In CNF,

there is a restriction on the number of symbols on the RHS of the production

- CNF has $A \rightarrow BC$ (variables) or $A \rightarrow a$ (terminal)

In GNF, there is no restriction in the number of symbols on the RHS BUT there is a restriction on the terminals and variables appearing on the RHS of the production

CNF → GNF

If a grammar is in GNF, we can easily convert it into PDA which can accept only CFL's.

Greibach normal form (GNF) [illegible]

A grammar $G = (V, T, P, S)$ is said to be in GNF if all the productions are of the form

$$A \to a\alpha$$

where $a \in T$ and $\alpha \in V^*$

i.e., first symbol in the RHS must be a terminal and it can be followed by zero or more variables.

Procedure

Step 1: Obtain the grammar in CNF

Step 2: Rename the non-terminals to $A_1, A_2, A_3, \ldots$

Step 3: Using the substitution method obtain the productions of the form

$$A_i \to A_j\alpha \text{ for } i < j$$

$$\alpha \in V^*$$

Step 4: After substitution, if a grammar has left-recursion, we should eliminate it

Step 5 → repeat step 3 and/or 4 till we get the grammar in GNF

Convert the following grammar to GNF. ②

Sol

$S \rightarrow AB1 \mid 0$

$A \rightarrow 00A \mid B$

$B \rightarrow 1A1$

- If grammar has ε-productions and unit-productions, first eliminate them
- Above grammar has no ε-production
- Remove the unit-production $A \rightarrow B$

$A \rightarrow 00A \mid B$ and $B \rightarrow 1A1$

$A \rightarrow 00A \mid 1A1$

- Resulting Grammar is

$S \rightarrow AB1 \mid 0$

$A \rightarrow 00A \mid 1A1$

$B \rightarrow 1A1$

- Convert into CNF
  - replace terminals by non-terminals

$S \rightarrow ABA_1 \mid 0$

$A \rightarrow A_0 A_0 A \mid A_1 A A_1$

$B \rightarrow A_1 A A_1$

- restrict # of variables on RHS to two

$S \to AD_1 | 0$ $D_1 = BA_1$

$A \to A_0D_2 | A_1D_3$ $D_2 = A_0A$

$A_1 = 1$ $D_3 = AA_1$

$A_0 = 0$

- Let us rename all the variables in an increasing order

$S = A_1$, $A = A_2$, $B = A_3$, $A_0 = A_4$, $A_1 = A_5$, $D_1 = A_6$, $D_2 = A_7$, $D_3 = A_8$

- Grammar can now be written as:

$A_1 \to A_2A_6 | 0$

$A_2 \to A_4A_7 | A_5A_8$

$A_3 \to A_5A_8$

$A_5 \to 1$

$A_4 \to 0$

$A_6 \to A_3A_5$

$A_7 \to A_4A_2$

$A_8 \to A_2A_5$

- Substitute values of terminals

$A_3 \to 1A_8$ (GNF)

$A_4 \to 0$ } GNF
$A_5 \to 1$ }

substitute these in A_2,

Consider A_2

$A_2 \rightarrow A_4A_7 \mid A_3A_8 = 0A_7 \mid 1A_8$ (4)

Consider A_1

$A_1 \rightarrow A_2A_6 \mid 0 = (0A_7 \mid 1A_8)A_6 \mid 0$

$= 0A_7A_6 \mid 1A_8A_6 \mid 0$] GNF

Consider A_6

$A_6 \rightarrow A_3A_5 = (A_3A_8)A_5 = 1A_8A_5$] GNF

Consider A_7

$A_7 \rightarrow A_4A_2 = 0A_2$] GNF

Consider A_8

$A_8 \rightarrow A_2A_5 = (0A_7 \mid 1A_8)A_5$

$= 0A_7A_5 \mid 1A_8A_5$] GNF

$G = (V, T, P, S)$

$V = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$

$T = \{0, 1\}$

$P = \{ A_1 \rightarrow 0A_7A_6 \mid 1A_8A_6 \mid 0$

$A_2 \rightarrow 0A_7 \mid 1A_8$

$A_3 \rightarrow 1A_8$

$A_4 \rightarrow 0$

$A_5 \rightarrow 1$

$A_5 \rightarrow 1A_8A_5$

$A_6 \rightarrow 0A_2$ & $A_8 \rightarrow 0A_7A_5 \mid 1A_8A_5$

S → Start $A_7 \rightarrow 0A_2$

A → BC
B → CA | b
C → AB | a into GNF

Let $A = A_1$ $B = A_2$ $C = A_3$

$A_1 \rightarrow A_2A_3$ — ①

$A_2 \rightarrow A_3A_1 | b$ — ②

$A_3 \rightarrow A_1A_2 | a$ — ③

① and ② are of the form

$A_i \rightarrow A_j\alpha$ for $i<j$

consider $A_3 \rightarrow$ ~~A_1A_2~~ and substitute value of A_1 from ①

$A_3 \rightarrow A_1A_2 | a$ = ~~$A_2A_3 | a$~~)

$= (A_2A_3)A_2 | a$

$= A_2A_3A_2 | a$

Substitute value of A_2 (first) from ②

$A_3 \rightarrow (A_3A_1 | b)A_3A_2 | a$

$= A_3A_1A_3A_2 | bA_3A_2 | a$.

$= A_3 \underbrace{A_1A_3A_2}_{\alpha} | b \underbrace{A_3A_2}_{\beta} | a$

has left recursion so eliminate it, $A \rightarrow \alpha A'$

$A \rightarrow A\alpha_i | \beta_i$
$A' \rightarrow \beta_i | \beta_i Z$
$Z \rightarrow \alpha_i | \alpha_i Z$

$A_3 \rightarrow bA_3A_2 \mid a \mid bA_3A_2Z \mid aZ$ } GNF (7)

$Z \rightarrow A_1A_3A_2 \mid A_1A_3A_2Z$

Consider A_2

$A_2 \rightarrow A_3A_1 \mid b$ substitute A_3 from above.

$= (bA_3A_2 \mid a \mid bA_3A_2Z \mid aZ)A_1 \mid b$

$= bA_3A_2A_1 \mid aA_1 \mid bA_3A_2ZA_1 \mid aZA_1 \mid b$ } GNF

Consider A_1

$A_1 \rightarrow A_2A_3$ substitute A_2 from above.

$A_1 = (bA_3A_2A_1 \mid aA_1 \mid bA_3A_2ZA_1 \mid aZA_1 \mid b)A_3$

$= bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2ZA_1A_3 \mid$
$aZA_1A_3 \mid bA_3$

Consider Z - productions

$Z \rightarrow A_1A_3A_2 \mid A_1A_3A_2Z$ substitute A_1

$Z \rightarrow (bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2ZA_1A_3 \mid aZA_1A_3 \mid bA_3)A_3A_2$

$\mid$

$(bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2ZA_1A_3 \mid aZA_1A_3 \mid bA_3)A_3A_2Z$

— multiply and rewrite

$G = (V, T, P, S)$

$S \to AA \mid 0$
$A \to SS \mid 1$ into GNF. (7)

$S = A_1 \quad A = A_2$

$A_1 = A_2 A_2 \mid 0$ —(1) → already in $A_i \to A_j \alpha \quad i<j$

$A_2 = A_1 A_1 \mid 1$ —(2)

So consider this

$A_2 \to A_1 A_1 \mid 1$ substitute A_1 from (1)

$A_2 \to (A_2 A_2 \mid 0) A_1 \mid 1$

$= A_2 A_2 A_1 \mid 0 A_1 \mid 1$

left recursive ⇒ eliminate it

$A_2 \to 0 A_1 \mid 1 \mid 0 A_1 Z \mid 1 Z$

$Z = A_2 A_1 \mid A_2 A_1 Z$

A_2 is in GNF

Consider A_1

$A_1 \to A_2 A_2 \mid 0$

$\to (0 A_1 \mid 1 \mid 0 A_1 Z \mid 1Z) A_2 \mid 0$

$A_1 = 0 A_1 A_2 \mid 1 A_2 \mid 0 A_1 Z A_2 \mid 1 Z A_2 \mid 0$

GNF

[illegible] Z productions

$Z = A_2 A_1 \mid A_2 A_1 Z$

$= (0A_1 \mid 1 \mid 0A_1 Z \mid 1Z) A_1 \mid$

$(0A_1 \mid 1 \mid 0A_1 Z \mid 1Z) A_1 Z$

$= 0A_1 A_1 \mid 1A_1 \mid 0A_1 Z A_1 \mid 1Z A_1 \mid$

$0A_1 A_1 Z \mid 1A_1 Z \mid 0A_1 Z A_1 Z \mid 1Z A_1 Z$

$\therefore G = (V, T, P, S)$

$V = (A_1, A_2, Z)$

$T = \{0, 1\}$

$P = \{ A_1 \rightarrow 0A_1 A_2 \mid 1A_2 \mid 0A_1 Z A_2 \mid 1Z A_2 \mid 0$

$A_2 \rightarrow 0A_1 \mid 1 \mid 0A_1 Z \mid 1Z$

$Z \rightarrow 0A_1 A_1 \mid 1A_1 \mid 0A_1 Z A_1 \mid 1Z A_1$

$\mid 0A_1 A_1 Z \mid 1A_1 Z \mid 0A_1 Z A_1 Z \mid 1Z A_1 Z$

$\}$

S is the start symbol.
